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Magnetization reversal through flipping solitons under the localized inhomogeneity

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Abstract

We study the nonlinear dynamics of a site-dependent Heisenberg ferromagnetic spin chain with Gilbert damping in the continuum limit and its associated dynamics which is governed by an inhomogeneous generalized higher order nonlinear Schrödinger equation. The effect of inhomogeneity was understood by carrying out a multiple perturbation analysis and the coupled evolution equations for the velocity and amplitude of the soliton were solved using the Jacobi elliptic function method. The evolution of the amplitude and velocity of the soliton leads to magnetization reversal via flipping of solitons in the ferromagnetic medium. Finally, we have also constructed the perturbed soliton solutions.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

The studies of an ultrafast magnetization reversal process in ferromagnetic media have become one of the most exciting topics in contemporary magnetism. They break new ground scientifically and explore length and time scales for tomorrow's magnetic technologies. The future development of magnetic recording technology, where the quest is for smaller magnetic bits and faster magnetic switching, depends to a large extent on the outcome of this research. The magnetization reversal process obtained through an understanding of the underlying magnetization dynamics is an important issue mainly because the dynamic process is nonlinear in nature. The magnetization reversal process is normally based on a coherent rotation of the magnetization and a propagation of domain walls in the presence of the magnetic

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field [1]. In that case, the switching of magnetization is accomplished by the domain wall motion which is a relatively slow process compared to a magnetization reversal by a coherent rotation [2]. One cannot rule out the possibility of magnetization reversal without applying external magnetic fields. Recently, it was shown that a dc pulse induces switching in magnetic materials with uniaxial anisotropy without having applied the external magnetic field [3]. It is proved that the fastest way to record a bit can be achieved by reversing the magnetization through precessional motion via the magnetic field and switching by spin currents which lies in the picosecond regime [4]. Much experimental evidence shows that the excitation of a ferromagnetic solid or film with an ultrashort laser pulse leads to demagnetization on a time scale of a few hundred femtosecond [5]. Recently, the fastest magnetic switching process has been achieved in the femtosecond regime through the LINAC electron pulse [6]. At present the magnetic device that responds to the external magnetic field is on the nanosecond time scale and it is observed that the magnetization reversal process through thermal activation is also in the nanoscale regime. In the conventional magnetic recording, the reversing field is applied antiparallel to the direction of the magnetization that limits the reversal speed to the nanosecond regime [7]. The magnetization switching achieved through other processes such as thermal activation, stress-induced anisotropy and by applying a high intense laser pulsed field has its own demerits like loss of information due to heating up of atoms. There are well-studied methods and processes that offer faster manipulation of the magnetization and even the complete switching between two well-defined magnetic states can be accomplished much faster. Most of the available results on the magnetization reversal process is based on experimental studies and numerical simulations, and analytical results are very limited [8–10]. Yet the understanding of the various physical processes on a faster time scale is presently incomplete and their understanding remains a challenge. In this respect, it has been recognized that the classical Landau–Lifshitz equation which governs the precessional motion of spins is a useful model to describe the fast magnetization process [11, 12]. The switching time of a magnetization reversal process mainly depends on the Gilbert damping present in the magnetic system. In principle, the fundamental limit of the switching speed via precession is given by a half of the precession period. The Landau–Lifshitz (LL) equation would yield the implausible result that the reversal time approaches zero as $\hat{\lambda} \longrightarrow \infty$, that is, the greater the damping the shorter the reversal time [13]. The LL equation would also yield the result that the reversal time is proportional to $\frac{1}{\hat{\lambda}}$. In an entirely different context, the LL equation corresponding to different magnetic interactions has been proved to be completely integrable, admitting soliton solutions in several cases [14–16]. For the past few decades, several attempts have been made to study the dynamics of different magnetic interactions such as bilinear isotropic exchange, single ion anisotropy due to the crystal field effect, inhomogeneity in the exchange interaction and interaction with the external magnetic field, etc, which have been identified as integrable models with localized spin excitations such as soliton, and domain wall under the different circumstances [17–19] in the classical continuum limit. The nonlinear dynamics of inhomogeneous systems have been widely investigated and expected to have many applications in the construction of magnetic memory devices, logic gates and so on. Furthermore, it has been proved that the inhomogeneous one dimensional system as well as radially symmetric spin system, the inhomogeneous compressible biquadratic Heisenberg ferromagnetic spin chain with harmonic lattice vibration, the inhomogeneous vortex filaments and the inhomogeneous exchange interactions are found to be integrable under certain conditions and exhibit nonlinear spin excitations in terms of solitons [20–23]. Also, it has been demonstrated that in [24] the site-dependent ferromagnetic spin chain with linear inhomogeneity admits the shape changing property during its evolution. This shape changing property can be exploited to reverse the magnetization without loss of energy, which may have potential applications in magnetic memory and recording devices. In some of the other contexts, it has been manifested that different types of nonlinear inhomogeneities have been shown to support soliton creation and annihilation in a site-dependent biquadratic ferromagnetic medium [25]. Thus, it has become increasingly important to investigate the magnetization reversal process by exploiting the localized coherent structure of solitons in ferromagnetic media.

In this paper, we demonstrate the magnetization reversal using solitons in a site-dependent ferromagnet under the influence of localized inhomogeneity. In section 2, we derive the equation of motion for a site-dependent ferromagnet in the presence of relativistic Gilbert damping. In section 3, we carry out the multiple scale perturbation method to derive the evolution equations for soliton parameters and demonstrate reversal of magnetization under the influence of tangent hyperbolic inhomogeneity. The paper is concluded in section 4.

2. Dynamics of a site-dependent bilinear ferromagnet

Most of the studies on magnetic spin chains have been based on the homogeneous Heisenberg Hamiltonian, where the exchange interaction coupling between the nearest-neighbour pair of spins is a single constant J or at most two constants as in the case of uniaxial anisotropy. But the presence of the magnetic defects introduced inhomogeneity in the exchange interaction. Generally, inhomogeneity in magnetic materials arises because of the following two factors. (i) If the distance between neighbouring magnetic atoms varies along the chain depending on the distance between the spins and the degree of overlapping of electronic wavefunction which varies from site to site. Thus, the interaction between the spins depends upon the site in the crystal lattice, which is known as the site-dependent interaction. This type of inhomogeneity occurs in charge transfer complexes TCNQ, $Ni(CN)_4$, organo-metallic insulators, TTF-bisdithiolenes and $Ni(Co)_4$ in which the characterizing inhomogeneous parameter alternates between two values as we move along the spin chain. (ii) If the atomic wavefunction itself varies from site to site, although the atoms themselves may equally be spaced. This type of inhomogeneity occurs when magnetic insulators were placed in a weak, static and inhomogeneous electric field. It can also be simulated by the deliberate introduction of imperfections (impurities or organic complexes) in the vicinity of a bond so as to alter the electronic wavefunctions without causing appreciable lattice distortion. For our model, the associated Landau–Lifshitz–Gilbert (LLG) equation can be written as

$$\vec{S}_t = \vec{S} \wedge \vec{F}_{\text{eff}} + \hat{\lambda}[\vec{F}_{\text{eff}} - (\vec{S}.\vec{F}_{\text{eff}})\vec{S}], \tag{1}$$

where $\vec{S} = (S^x, S^y, S^z)$ represents the classical three-component spin vector and $\vec{S}^2 = 1$. The first term in equation (1) describes the precessional motion of a magnetization vector or spin vector $\vec{S}(x, t)$ about $\vec{F}_{\text{eff}}$ and the second term represents the phenomenological damping parameter. The phenomenological (Gilbert) damping parameter which causes $\vec{S}(x, t)$ gets relaxed along or anti-parallel to $\vec{F}_{\text{eff}}$ depending on the nature of the sign of $\hat{\lambda}$. Conventionally, $\hat{\lambda}$ is identified as $\alpha\gamma$, where α is the dimensionless Gilbert damping parameter and γ is the gyromagnetic ratio. In equation (1), the $\vec{F}_{\text{eff}}$ contribution may come from the exchange interaction, crystalline anisotropy, magnetostatic self-energy, external magnetic fields, thermal fluctuations and so on. In particular, the effective field $\vec{F}_{\text{eff}}$ contribution due to the site-dependent bilinear exchange interaction in the classical continuum limit is given by

$$\vec{F}_{\text{eff}} = h_x\vec{S}_x + h\vec{S}_{xx}. \tag{2}$$

Making use of equation (2) in equation (1),

$$\vec{S}_t = \vec{S}\wedge(h_x\vec{S}_x + h\vec{S}_{xx}) + \hat{\lambda}[h_x\vec{S}_x + h\vec{S}_{xx} - h(\vec{S}.\vec{S}_{xx})\vec{S}]. \tag{3}$$

The function $h = h(x)$ in the effective field $\vec{F}_{\rm eff}$ as in equation (2) determines the inhomogeneity along the spin chain. In order to understand the evolution of nonlinear spin dynamics of a site-dependent bilinear ferromagnetic spin chain, the spin vector $\vec{S}(x,t)$ is mapped on the unit tangent $\vec{e}_1(x,t)$ of the moving helical space curve in E^3 through the well-known procedure in the classical differential geometry and the unit normal $\vec{e}_2(x,t)$ and bi-normal $\vec{e}_3(x,t)$ of the curve form a local coordinate system with the origin O' in E^3 [26]. The variation along the space curve is given by the usual Serret–Frenet (SF) equations as

$$\begin{pmatrix}\vec{e}_{1x}\\ \vec{e}_{2x}\\ \vec{e}_{3x}\end{pmatrix} = \begin{pmatrix}0 & \kappa & 0\\ -\kappa & 0 & \tau\\ 0 & -\tau & 0\end{pmatrix}\begin{pmatrix}\vec{e}_1\\ \vec{e}_2\\ \vec{e}_3\end{pmatrix}. \tag{4}$$

Here, $\kappa = (\vec{e}_{1x}.\vec{e}_{1x})^{1/2}$ is the curvature and $\tau = \kappa^{-2}\vec{e}_1.(\vec{e}_{1x} \times \vec{e}_{1xx})$ is the torsion of the space curve. In view of the above identifications, the evolution of $\vec{e}_1$ of the trihedron using equation (3) is given by

$$\vec{e}_{1t} = -(h\kappa\tau + \hat{\lambda}(h\kappa)_x)\vec{e}_2 + ((h\kappa)_x + \hat{\lambda}h\kappa\tau)\vec{e}_3. \tag{5}$$

The evolution of the trihedron $\vec{e}_i$, where $i = 1, 2, 3$, can be evaluated using equations (4) and (5) and is written as

$$\begin{pmatrix}\vec{e}_{1t}\\ \vec{e}_{2t}\\ \vec{e}_{3t}\end{pmatrix} = \begin{pmatrix}0 & \omega_3 & -\omega_2\\ -\omega_3 & 0 & \omega_1\\ \omega_2 & -\omega_1 & 0\end{pmatrix}\begin{pmatrix}\vec{e}_1\\ \vec{e}_2\\ \vec{e}_3\end{pmatrix}, \tag{6}$$

where

$$\begin{aligned}\omega_1 &= \frac{(h\kappa)_{xx}}{\kappa} - h\tau^2 - \frac{\hat{\lambda}}{\kappa}(2\tau(h\kappa)_x + h\kappa\tau_x),\\ \omega_2 &= -(h\kappa)_x + \hat{\lambda}h\kappa\tau,\\ \omega_3 &= -h\kappa\tau - \hat{\lambda}(h\kappa)_x.\end{aligned}$$

The compatibility conditions are $(\vec{e}_{ix})_t = (\vec{e}_{it})_x$, where $i = 1, 2, 3$. The SF equations (4) and (6) lead to the evolution of curvature $\kappa(x,t)$ and torsion $\tau(x,t)$ of the space curve:

$$\kappa_t = -\tau(h\kappa)_x + \hat{\lambda}h\kappa\tau^2 - [h\kappa\tau + \hat{\lambda}(h\kappa)_x]_x, \tag{7a}$$

$$\tau_t = \kappa[(h\kappa)_x - \hat{\lambda}h\kappa\tau] - \left[h\tau^2 - \frac{(h\kappa)_{xx}}{\kappa} + \frac{\hat{\lambda}}{\kappa}(2\tau(h\kappa)_x + h\kappa\tau_x)\right]_x. \tag{7b}$$

The energy and current densities of the undamped ($\hat{\lambda} = 0$) inhomogeneous spin chain are given by

$$E(x,t) = \frac{h}{2}\left(\frac{\partial\vec{S}}{\partial x}\right).\left(\frac{\partial\vec{S}}{\partial x}\right) = \frac{h}{2}\kappa^2(x,t), \tag{8}$$

$$I(x,t) = h^2\vec{S}.\left(\frac{\partial\vec{S}}{\partial x} \times \frac{\partial^2\vec{S}}{\partial x^2}\right) = h^2\kappa^2(x,t)\tau(x,t). \tag{9}$$

Thus, the torsion and curvature of the space curve are related to the energy and current densities of the spin system through the above equations (8) and (9) and hence the spin dynamics is equivalently represented in terms of the curvature and torsion of the space curve. In order to identify the set of coupled equations (7) with a more standard nonlinear partial differential equation, we make the following complex transformation:

$$q(x,t) = \frac{1}{2}\kappa(x,t)\exp\left\{\mathrm{i}\int_{-\infty}^{x}\tau(x',t)\,\mathrm{d}x'\right\}, \tag{10}$$

and obtain the following damped inhomogeneous nonlinear Schrödinger (DINLS) equation:

$$\mathrm{i}q_t + (hq)_{xx} + 2h|q|^2q + 2q\int_{-\infty}^{x} h_{x'}|q|^2\,\mathrm{d}x' - \mathrm{i}\hat{\lambda}\left[(hq)_{xx} - 2q\int_{-\infty}^{x} h(qq_x^* - q^*q_x)\,\mathrm{d}x'\right] = 0, \tag{11}$$

when $h(x)$ is a linear function of x and $\hat{\lambda} = 0$; equation (11) is completely reduced to the inhomogeneous nonlinear Schrödinger equation and admits soliton solutions.

3. Effect of nonlinear inhomogeneity on the spin soliton

The results of singularity structure analysis on equation (11) show that it becomes completely integrable and the elementary spin excitations can be expressed in terms of solitons for specific choice of parameters and only when the exchange inhomogeneity appears in the form of a linear function. When the inhomogeneity is a invariant quantity i.e. $h(x)$ = constant, the dynamics of a one-dimensional classical continuum isotropic Heisenberg ferromagnetic spin system in the presence of a weak relativistic interaction is studied by Daniel *et al* [27]. He observed that when $h(x)$ = constant and characterizing the non-varying bilinear exchange interaction, the amplitude of the soliton asymptotically decreases to zero while the velocity of the soliton monotonically increases to attain a constant value, without showing any sign of reversal. Also, recently the dynamics of an inhomogeneous Heisenberg ferromagnetic spin chain with Gilbert damping has been found to exhibit the shape changing property under the linear inhomogeneity [24]. Further, it is also proved that the nonlinear inhomogeneity is a good candidate for inducing the magnetization reversal through flipping of solitons in the ferromagnetic media with higher order exchange interactions [28]. Keeping the above in mind, the natural question arises as to what will be the effect of a localized nonlinear inhomogeneity on the spin soliton. In this section, we try to find answer for this question by carrying out a multiple scale perturbation by considering the tangent hyperbolic localized inhomogeneity and by treating the inhomogeneity as perturbation on the spin soliton. At the out set, we substitute

$$h(x) = h_0 + \lambda h_1(x), \tag{12}$$

where h_0 and λ are a constant and a small parameter respectively and $h_1(x)$ is a nonlinear function of x in equation (12). After a suitable rescaling and a redefinition of $\hat{\lambda}$ as $\hat{\lambda} = \lambda\gamma$, where γ is damping parameter, the equation reads

$$\mathrm{i}q_t + q_{xx} + 2|q|^2q + \lambda\Bigg[(h_1q)_{xx} + 2h_1|q|^2q + 2q\int_{-\infty}^{x} h_{x'}|q|^2\,\mathrm{d}x' \\ - \mathrm{i}\gamma\left\{q_{xx} - 2q\int_{-\infty}^{x}(qq_{x'}^* - q^*q_{x'})\,\mathrm{d}x'\right\}\Bigg] = 0. \tag{13}$$

We study the effect of localized inhomogeneity by treating terms proportional to λ in equation (13) using the perturbation method as laid down by Kodama and Ablowitz [29]. When $\lambda = 0$, equation (13) reduces to the completely integrable cubic NLS equation which admits the envelope one-soliton solution in the form

$$q = \eta\,\mathrm{sech}\,\eta(\theta - \theta_0)\exp[\mathrm{i}\xi(\theta - \theta_0) + \mathrm{i}(\sigma - \sigma_0)], \tag{14}$$

where $\theta_t = -2\xi$, $\theta_x = 1$, $\sigma_t = \eta^2 + \xi^2$ and $\sigma_x = 0$. η and ξ are related to the scattering parameter of the inverse scattering transform (IST) analysis. Now, we write η (amplitude), ξ (velocity), θ, θ_0 and σ_0 as functions of a new time scale $T = \lambda t$, and $q = \hat{q}(\theta, T; \lambda)\exp[\mathrm{i}\xi(\theta - \theta_0) + \mathrm{i}(\sigma - \sigma_0)]$. Under the assumption of quasi-stationary, we then expand $\hat{q}$ in terms of λ as $\hat{q}(\theta, T; \lambda) = \hat{q}_0(\theta, T) + \lambda\hat{q}_1(\theta, T) + \cdots$, where $\hat{q}_0 = \eta\,\mathrm{sech}\,\eta(\theta - \theta_0)$ and making use of equation (14) in equation (13) at O(λ), we obtain

$$-\eta^2\hat{q}_1 + \hat{q}_{1\theta\theta} + 2\hat{q}_0^2\hat{q}_1^* + 4\hat{q}_0^2\hat{q}_1 = F_1(\hat{q}_0). \tag{15}$$

After substituting $\hat{q}_1 = \hat{\phi}_1 + \mathrm{i}\hat{\psi}_1$ in equation (15), where $\hat{\phi}_1$ and $\hat{\psi}_1$ are real functions, we obtain

$$L_1\hat{\phi}_1 = -\eta^2\hat{\phi}_1 + \hat{\phi}_{1\theta\theta} + 6\hat{q}_0^2\hat{\phi}_1 = \Re F_1(\hat{q}_0), \tag{16a}$$

$$L_2\hat{\psi}_1 = -\eta^2\hat{\psi}_1 + \hat{\psi}_{1\theta\theta} + 2\hat{q}_0^2\hat{\psi}_1 = \Im F_1(\hat{q}_0), \tag{16b}$$

where L_1 and L_2 are the self-adjoint operators. As $\hat{q}_{0\theta}$ and $\hat{q}_0$ are the solutions of the homogeneous parts of equations (16) for $\hat{\phi}_1$ and $\hat{\psi}_1$ respectively, the secularity conditions yield

$$\int_{-\infty}^{\infty} \hat{q}_{0\theta}\Re F_1\,\mathrm{d}\theta = 0 \qquad \text{and} \qquad \int_{-\infty}^{\infty} \hat{q}_0\Im F_1\,\mathrm{d}\theta = 0, \tag{17}$$

where

$$\Re F_1(\hat{q}_0) = h_1\xi^2\hat{q}_0 - 2h_1\hat{q}_0^3 + (\xi_T(\theta - \theta_0) - \xi\theta_{0T} - \sigma_{0T})\hat{q}_0 - (h_1\hat{q}_0)_{\theta\theta} - 2\gamma\xi\hat{q}_{0\theta} \\ - 2\hat{q}_0\int_{-\infty}^{\theta} h_{1\theta}\hat{q}_0^2\mathrm{d}\theta - 4\gamma\xi\hat{q}_0\int_{-\infty}^{\theta}\hat{q}_0^2\,\mathrm{d}\theta, \tag{18}$$

$$\Im F_1(\hat{q}_0) = \gamma\hat{q}_{0\theta\theta} - \hat{q}_{0T} - 2\xi h_1\hat{q}_{0\theta} - 2\xi h_{1\theta}\hat{q}_0 - \gamma\xi^2\hat{q}_0 - 2\gamma\hat{q}_0\int_{-\infty}^{\theta}(\hat{q}_0\hat{q}_{0\theta}^* - \hat{q}_0^*\hat{q}_{0\theta})\,\mathrm{d}\theta. \tag{19}$$

In order to evaluate the integrations in equation (17), we need to provide the explicit form of the inhomogeneity h_1 in equations (18) and (19).

3.1. Magnetization reversal

The secularity conditions (17), on substituting $h_1(x) = \hat{\alpha}\tanh\eta(\theta - \theta_0)$, yield a set of coupled evolution equations for the amplitude and velocity of the soliton. On evaluating the integrals and after rescaling $T \longrightarrow -\frac{T}{3}$, $\alpha_1 = 2\hat{\alpha}$ and $\alpha_2 = 2\gamma$, the derivatives of the evolution equations read

$$\xi_{TT} = 4\left(\alpha_1^2 - 3\alpha_2^2\right)\xi^3\eta^2 - \alpha_1\alpha_2(22\xi^2\eta^3 - 3\xi^4\eta + \eta^5) - 8\alpha_1^2\xi\eta^4, \tag{20}$$

$$\eta_{TT} = \left(3\alpha_2^2 - 2\alpha_1^2\right)\eta^5 + 10\alpha_1^2\xi^2\eta^3 + 24\alpha_1\alpha_2\xi^3\eta^2 + 9\alpha_2^2\eta\xi^4. \tag{21}$$

In order to understand the effect of inhomogeneity on the velocity and amplitude of the soliton, we solve equations (20) and (21), by employing the Jacobi elliptic function method. Further, we try to find answer for the question whether varying the exchange interaction can induce the magnetization reversal process in an inhomogeneous ferromagnetic system. Recently, the Jacobi elliptic sine, cosine function expansion method and the third kind of Jacobi elliptic function method have been proposed as powerful methods to construct a new kind of periodic solution of nonlinear wave equations [30, 31]. It shows that under the limiting condition, the nonlinear evolution equation exhibits shock-wave solutions or soliton solutions. Suppose we have a nonlinear evolution equation

$$N\left(\phi, \frac{\mathrm{d}\phi}{\mathrm{d}\tau}, \frac{\mathrm{d}^2\phi}{\mathrm{d}\tau^2}, \frac{\mathrm{d}^3\phi}{\mathrm{d}\tau^3}, \ldots\right) = 0, \tag{22}$$

where ϕ is expressed as a finite series of the Jacobi elliptic sine function $sn(\tau|k)$, similarly, the Jacobi elliptic cosine function $cn(\tau|k)$, or the Jacobi elliptic functions of third kind $dn(\tau|k)$,

$$\phi(\tau) = \sum_{i=0}^{n} a_i sn^i(\tau|k), \tag{23}$$

where a_i are constants to be determined later and the higher degree is

$$O[\phi(\tau)] = n. \tag{24}$$

Therefore, the highest degree of $\frac{\mathrm{d}^p\phi}{\mathrm{d}\tau^p}$ is taken as

$$O\left[\frac{\mathrm{d}^p\phi}{\mathrm{d}\tau^p}\right] = n + p, \qquad p = 0, 1, 2, \ldots, \tag{25a}$$

$$O\left[\phi^q \frac{\mathrm{d}^p\phi}{\mathrm{d}\tau^p}\right] = (q+1)n + p, \qquad p, q = 0, 1, 2, \ldots. \tag{25b}$$

Thus, in order to fix 'n' in equation (23) we have to balance the highest order derivative and nonlinear term in equation (22). Substituting equation (23) into equation (22) and equating to zero the coefficient of all powers of $sn(\tau|k)$, $cn(\tau|k)$ and $dn(\tau|k)$ yields a set of algebraic equations for a_i. Finally, by inserting each solution of this set of algebraic equations into equation (23), the exact solution can be obtained. Now assuming that equations (20) and (21) have the solution of the form of

$$\xi(T) = \sum_{i=0}^{n} a_i sn^i(T|k), \tag{26a}$$

$$\eta(T) = \sum_{j=0}^{m} b_j sn^j(T|k), \tag{26b}$$

where k is the modulus of the elliptic functions which represents the periodicity of the function and balancing the higher derivative terms with nonlinear terms, we obtain $n + 2 = 4n + m$ and $m + 2 = 3n + 2m$. On solving for n and m we obtain $n = 1$ and $m = -1$. Then using equations (27) into equations (20) and (21), we obtain a system of algebraic equations in terms of a_0, a_1, b_0, b_{-1}, α_1 and α_2 as presented in appendix A. Now solving the above system of algebraic equations using symbolic computation, we obtain the following solutions:

$$\xi(T) = a_0 + a_1 sn(T|k), \tag{27a}$$

$$\eta(T) = b_0 + b_{-1} sn(T|k)^{-1}, \tag{27b}$$

where

$$a_1 = -\frac{12\alpha_1\alpha_2 a_0^3 + 15\alpha_1^2 b_0 a_0^2 - 10\alpha_1^2 b_0^3 + 15\alpha_2^2 b_0^3}{10 b_{-1}\alpha_1^2 a_0}, \tag{28a}$$

$$b_{-1} = \frac{1}{\sqrt{5b_0^2(3\alpha_2^2 - 2\alpha_1^2) + 5\alpha_1^2 a_0^2}}, \tag{28b}$$

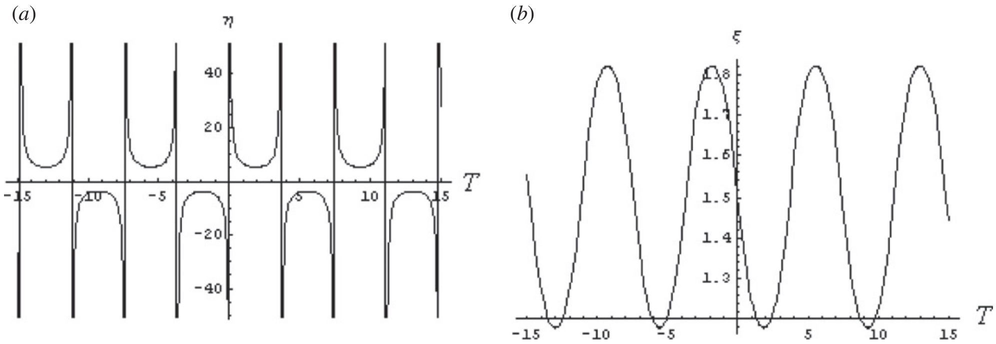

Figure 1. Evolution of (*a*) the amplitude and (*b*) the velocity of the soliton with $\hat{\alpha} = 0.81$, $\gamma = 0$, $a_0 = 1.5$, $b_0 = 0.8$ and $k = 0.5$.

and

$$a_1 = -\frac{b_0\left(27\alpha_2^2 a_0^2 + 36\alpha_1\alpha_2 b_0 a_0 + 5\alpha_1^2 b_0^2\right)}{6\alpha_2 b_{-1}(4\alpha_1 b_0 + 3\alpha_2 a_0)}, \tag{28c}$$

$$b_{-1} = \frac{\sqrt{5b_0^2\left(3\alpha_2^2 - 2\alpha_1^2\right) + 5\alpha_1^2 a_0^2}}{5b_0^2\left(2\alpha_1^2 - 3\alpha_2^2\right) - 5\alpha_1^2 a_0^2}. \tag{28d}$$

We try to demonstrate magnetization reversal via soliton flip by plotting the amplitude η and the velocity ξ of the soliton from equations (27) by choosing the parameters $\hat{\alpha} = 0.81$, $a_0 = 1.5$, $b_0 = 0.8$ and $k = 0.5$ in figure 1. From the figures, for the undamped case, we observe that as time passes the velocity and amplitude of the soliton increases and after reaching a maximum value the soliton suddenly flips leading to magnetization reversal, and the soliton starts moving in the opposite direction. Figure 1(*a*) depicts the amplitude evolution of the soliton and implies that the flipping of the soliton occurs periodically and continues indefinitely. However, the evolution of velocity of the soliton depicts only a marginal reversal as shown in figure 1(*b*). It is also observed that when the soliton amplitude changes from positive (negative) to negative (positive), it slowly moves either forward or backward. The amplitude of the soliton can be tuned to have marginal reversal as shown in figure 2. The velocity of the soliton can also be tuned by the damping parameter such that the soliton may move either forward or backward very fast, as depicted in figure 2. The velocity of the soliton shows dramatic turns at the points when it reverses or switches. The magnetization reversal through soliton flipping lies in the nanoscale regime. In our analysis, it is found that in the absence of damping, the soliton takes 7.5 ns to complete one cycle. More interestingly, it is also found that when we increase the value of the Gilbert damping parameter, the switching time is reduced to 5 ns. Moreover, we attempt to solve the evolution equations for soliton parameters numerically using the fourth-order Runge–Kutta method coupled with a time domain integration scheme and investigate the magnetization dynamics under the influence of localized inhomogeneity on the Heisenberg ferromagnet with Gilbert damping. We have deduced equations (20) and (21) after numerically solving for the amplitude $\eta(T)$ and the velocity $\xi(T)$ of the soliton by choosing the initial value of $\eta(T) = \eta(0) = 0.9$, $\xi(T) = \xi(0) = -0.3$ and $\hat{\alpha} = 0.5$ with a step size $h = 0.1$. We have compared these results of figures 3 and 4 with the analytical results and it is found that the numerical results are in close agreement with the analytical results.

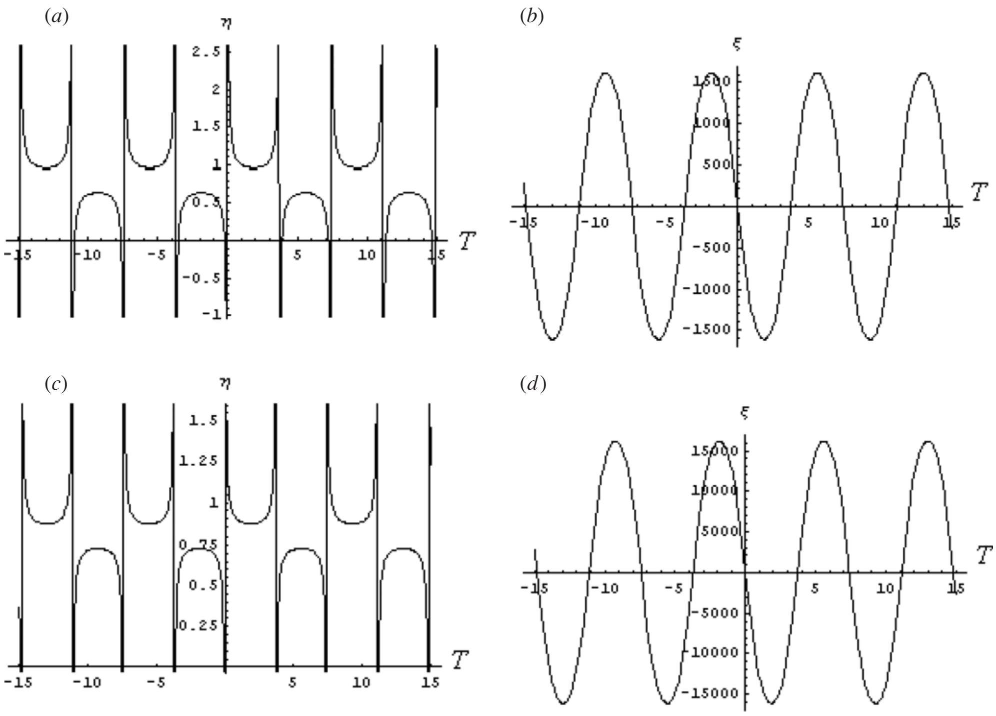

Figure 2. Evolution of (*a*, *c*) the amplitude and (*b*, *d*) the velocity of the soliton with $\hat{\alpha} = 0.81$, $a_0 = 1.5$, $b_0 = 0.8$ and $k = 0.5$. (*a*) and (*b*) for $\gamma = 1$ and (*c*) and (*d*) for $\gamma = 2.25$.

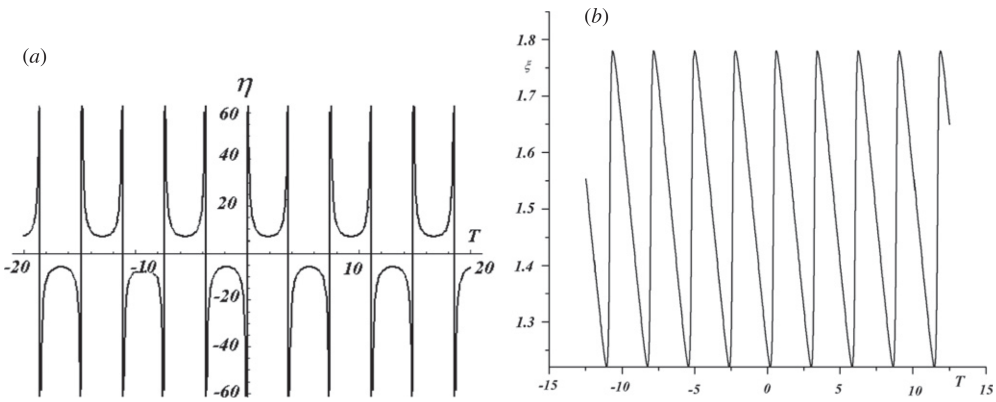

Figure 3. The evolution of numerical solutions for (*a*) the amplitude and (*b*) the velocity of the soliton with $\eta(0) = 0.9$, $\xi(0) = -0.3$, $\hat{\alpha} = 0.5$ and $\gamma = 0$.

3.2. Perturbed solitons

The perturbed soliton can be constructed by solving equation (16) for $\hat{\phi}_1$ and $\hat{\psi}_1$. The homogeneous part of equation (16*a*) admits the following two particular solutions:

$$\phi_{11} = \operatorname{sech}\eta(\theta-\theta_0)\tanh\eta(\theta-\theta_0), \tag{29a}$$

$$\phi_{12} = -\frac{1}{\eta}\left[\operatorname{sech}\eta(\theta-\theta_0) - \frac{3}{2}\eta(\theta-\theta_0)\phi_{11} - \frac{1}{2}\tanh\eta(\theta-\theta_0)\sinh\eta(\theta-\theta_0)\right]. \tag{29b}$$

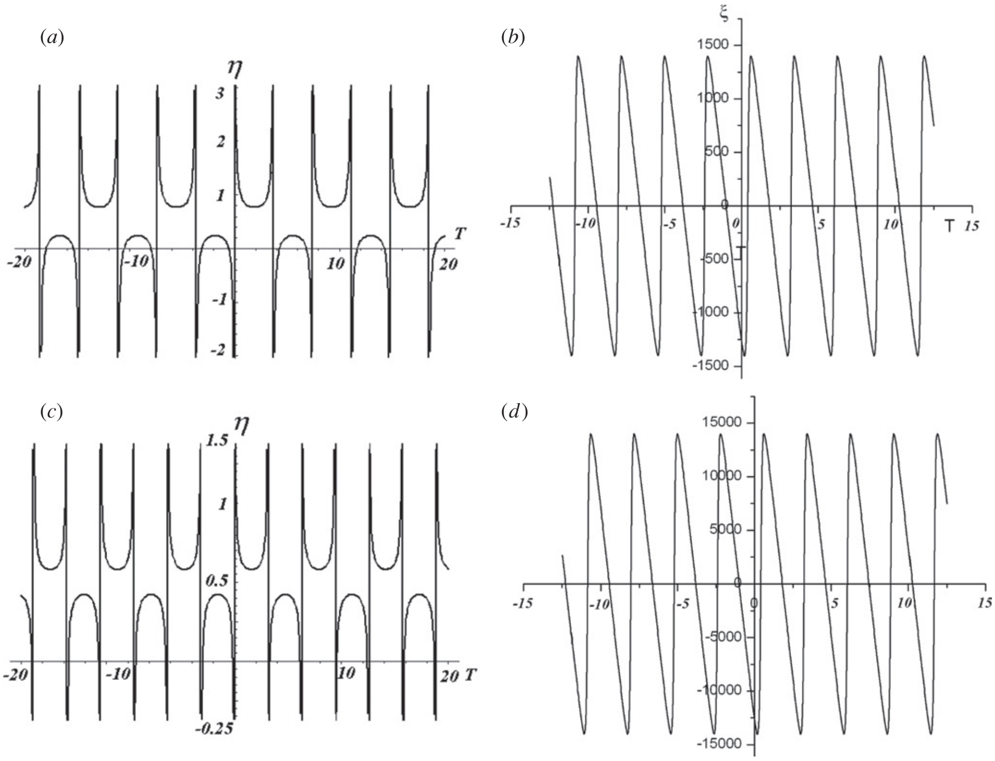

Figure 4. The evolution of numerical solutions for (*a*, *c*) the amplitude and (*b*, *d*) the velocity of the soliton with $\eta(0) = 0.9$, $\xi(0) = -0.3$ and $\hat{\alpha} = 0.5$. (*a*) and (*b*) for $\gamma = 1$ and (*c*) and (*d*) for $\gamma = 2.25$.

Knowing the two particular solutions, the general solution can then be obtained using the formula

$$\hat{\phi}_1 = \delta_1\phi_{11} + \delta_2\phi_{12} - \phi_{11}\int_{-\infty}^{\theta}\phi_{12}\Re F_1\,\mathrm{d}\theta' + \phi_{12}\int_{-\infty}^{\theta}\phi_{11}\Re F_1\,\mathrm{d}\theta', \tag{30}$$

where δ_1 and δ_2 are the arbitrary constants. Using ϕ_{11}, ϕ_{12} and $\Re F_1$ in equation (30) and after evaluating the integrals with lengthy algebra (for the details of calculation, please see appendix B), we write the general solution for $\hat{\phi}_1$ as

$$\begin{aligned}\hat{\phi}_1 = {}& \left[\delta_1 + \frac{3}{2}\delta_2\Theta + \frac{2}{3}\hat{\alpha}\eta\ln\cosh\eta\Theta - \frac{1}{4}\xi_T\Theta^2 - \frac{2}{3}\frac{\hat{\alpha}\xi^2}{\eta^2} - \frac{3}{4}\frac{\xi_T}{\eta^2} + \frac{1}{2}(\xi\theta_{0T} + \sigma_{0T})\Theta\right.\\ &\left. + \frac{4}{3}\gamma\xi + \hat{\alpha}\eta\right]\operatorname{sech}\eta\Theta\tanh\eta\Theta + \left[-\frac{\delta_2}{\eta} + \left(\frac{2}{3}\gamma\xi\eta + \frac{1}{3}\hat{\alpha}\eta^2 - \frac{1}{3}\hat{\alpha}\xi^2\right)\Theta\right.\\ &\left. - \frac{1}{2\eta}(\xi\theta_{0T} + \sigma_{0T})\right]\operatorname{sech}\eta\Theta - \frac{3}{2\eta}(\xi\theta_{0T} + \sigma_{0T})\operatorname{sech}^3\eta\Theta\\ & + \left[\frac{1}{6\eta^2}\left(\frac{3}{2}\xi_T + \hat{\alpha}\xi^2\eta - \hat{\alpha}\eta^3 - 2\gamma\xi\eta^2\right) + \frac{1}{2\eta}\delta_2\tanh\Theta\right]\sinh\Theta.\end{aligned} \tag{31}$$

We remove the secular terms, which make the solution unbounded by choosing the arbitrary constant

$$\delta_2 = -2(\xi\theta_{0T} + \sigma_{0T}). \tag{32}$$

Further using the boundary conditions

$$\hat{\phi}_1(0)|_{\theta_0=0} = 0, \qquad \hat{\phi}_{1\theta}(0)|_{\theta_0=0} = 0, \tag{33}$$

we obtain

$$\delta_1 = \frac{1}{\eta^2}\left(\frac{1}{4}\xi_T + \frac{2}{3}\hat{\alpha}\eta^2 - \frac{4}{3}\gamma\xi\eta^2 - \hat{\alpha}\eta^3\right). \tag{34}$$

Using equations (32) and (34), the explicit form of $\hat{\phi}_1$ can be constructed. In the similar way, $\hat{\psi}_1$ can also be evaluated using the two particular solutions corresponding to the homogeneous part of equation (16*b*) for $\hat{\psi}_1$ which are of the form

$$\psi_{11} = \operatorname{sech}\eta(\theta - \theta_0), \tag{35a}$$

$$\psi_{12} = \frac{1}{2\eta}[\eta(\theta - \theta_0)\operatorname{sech}\eta(\theta - \theta_0) + \sinh\eta(\theta - \theta_0)]. \tag{35b}$$

Knowing the two particular solutions, the general solution can then be obtained using the formula

$$\hat{\psi}_1 = \delta_3\psi_{11} + \delta_4\psi_{12} - \psi_{11}\int_{-\infty}^{\theta}\psi_{12}\Im F_1\,\mathrm{d}\theta' + \psi_{12}\int_{-\infty}^{\theta}\psi_{11}\Im F_1\,\mathrm{d}\theta', \tag{36}$$

where δ_3 and δ_4 are the arbitrary constants. Using ψ_{11}, ψ_{12} and $\Im F_1$ in equation (36) and after evaluating the integrals (for the details of calculation, please see appendix B), we write the general solution for $\hat{\psi}_1$ as

$$\begin{aligned}\hat{\psi}_1 = \Bigg[&\delta_3 + \frac{1}{2}\delta_4\Theta - \frac{1}{3}\gamma\eta - \frac{2}{3}\hat{\alpha}\xi - \frac{1}{2}\eta\Theta\Theta_T - \frac{2}{3}(\gamma\eta + 2\hat{\alpha}\xi)\ln\cosh\eta\Theta\\ &+ \left(\frac{1}{3}\hat{\alpha}\xi\eta^2 + \frac{1}{6}\gamma\eta^3 + \frac{1}{2}\gamma\xi^2\eta\right)\Theta^2\Bigg]\operatorname{sech}\eta\Theta\\ &- \left[\frac{2}{3}\hat{\alpha}\xi\eta + \frac{1}{3}\gamma\eta^2 + \gamma\xi^2\right]\Theta\operatorname{sech}\eta\Theta\tanh\eta\Theta\\ &+ \left[\frac{1}{2\eta}\delta_4 - \left(\frac{1}{4\eta^2}\eta_T + \frac{1}{6}\gamma\eta + \frac{1}{2}\gamma\xi^2 + \frac{1}{3}\hat{\alpha}\xi\right)\right]\tanh\Theta\sinh\Theta.\end{aligned} \tag{37}$$

We remove the secular terms, which make the solution unbounded by choosing the arbitrary constant

$$\delta_4 = 0. \tag{38}$$

Further using the boundary conditions

$$\hat{\psi}_1(0)|_{\theta_0=0} = 0; \qquad \hat{\psi}_{1\theta}(0)|_{\theta_0=0} = 0, \tag{39}$$

we obtain

$$\delta_3 = \tfrac{1}{3}(2\hat{\alpha}\xi + \gamma\eta). \tag{40}$$

Using equations (38) and (40), the explicit form of $\hat{\psi}_1$ can be constructed. Having obtained the explicit form of ξ, η, $\hat{\phi}_1$ and $\hat{\psi}_1$, we can easily construct the perturbed soliton solution $\hat{q}_1$ through the relation $\hat{q}_1 = \hat{\phi}_1 + \mathrm{i}\hat{\psi}_1$. The solution for $\hat{q}_1$ is

$$\begin{aligned}\hat{q}_1 = \Bigg[&\frac{1}{3}\frac{\hat{\alpha}\xi^2}{\eta} - \frac{1}{3}\hat{\alpha}\eta - \frac{2}{3}\gamma\xi + \frac{2}{3}\hat{\alpha}\eta\ln\cosh\eta\Theta - \left(\frac{1}{3}\gamma\xi\eta^2 + \frac{1}{6}\hat{\alpha}\eta^3 - \frac{1}{6}\hat{\alpha}\xi^2\eta\right)\Theta^2\\ &+ \frac{1}{2}(\xi\theta_{0T} + \sigma_{0T})\Theta\Bigg]\operatorname{sech}\eta\Theta\tanh\eta\Theta + \left[\left(\frac{2}{3}\gamma\xi\eta + \frac{1}{3}\hat{\alpha}\eta^2 - \frac{1}{3}\hat{\alpha}\xi^2\right)\Theta\right.\end{aligned}$$

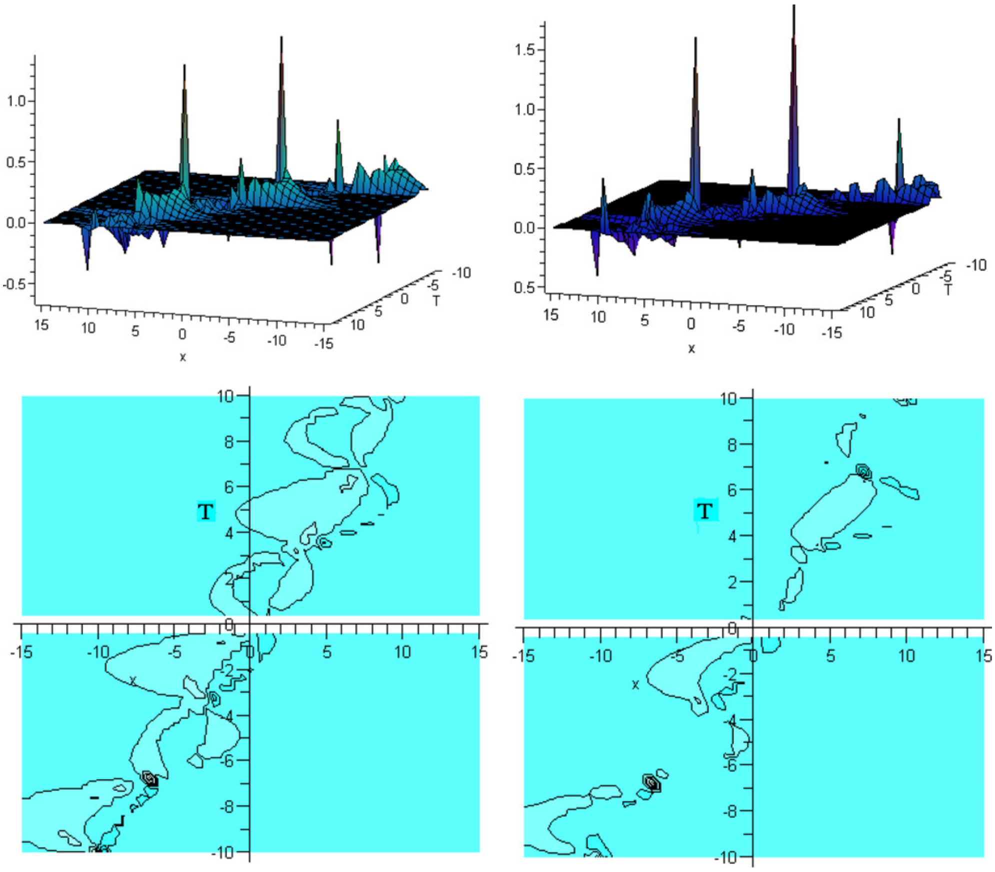

Figure 5. Real part of the perturbed soliton and its contour plots with $\hat{\alpha} = 0.7$, $\beta = 0.1$, $\delta = 0.5$, $a_0 = 0.5$, $b_0 = 0.3$ and $k = 0.5$ (i) $\gamma = 0$ (ii) $\gamma = 0.4$

$$
\begin{aligned}
&-\frac{1}{2\eta}(\xi\theta_{0T} + \sigma_{0T})\Big]\operatorname{sech}\eta\Theta - \frac{3}{2\eta}(\xi\theta_{0T} + \sigma_{0T})\operatorname{sech}^3\eta\Theta \\
&-\mathrm{i}\left\{\left[\frac{2}{3}\hat{\alpha}\xi\eta + \frac{1}{3}\gamma\eta^2 + \gamma\xi^2\right]\Theta\operatorname{sech}\eta\Theta\tanh\eta\Theta\right. \\
&-\left[\left(\frac{1}{3}\hat{\alpha}\xi\eta^2 + \frac{1}{6}\gamma\eta^3 + \frac{1}{2}\gamma\xi^2\eta\right)\Theta^2 - \frac{1}{2}\eta\Theta\Theta_T\right. \\
&\left.\left.-\frac{2}{3}(\gamma\eta + 2\hat{\alpha}\xi)\ln\cosh\eta\Theta\right]\operatorname{sech}\eta\Theta\right\}, \qquad (41)
\end{aligned}
$$

where $\Theta = \theta - \theta_0$. We have plotted the real and imaginary parts of the perturbed soliton by choosing the values of the parameters $\hat{\alpha} = 0.7$, $\beta = 0.1$, $\delta = 0.5$, $a_0 = 0.5$, $b_0 = 0.3$ and $k = 0.5$ in figures 5 and 6. Both the figures demonstrate the magnetization reversal via soliton flip which occurs periodically. This implies that when the soliton moves along the ferromagnetic spin chain, the presence of localized inhomogeneity in the varying bilinear exchange triggers switching through flipping of solitons. In the contour plots, figures 5(*b*) and 6(*b*), the bigger loops represent the unflipped states of the soliton and the smaller connecting loops represent the flipped (reversed) states of soliton. When the value of the damping

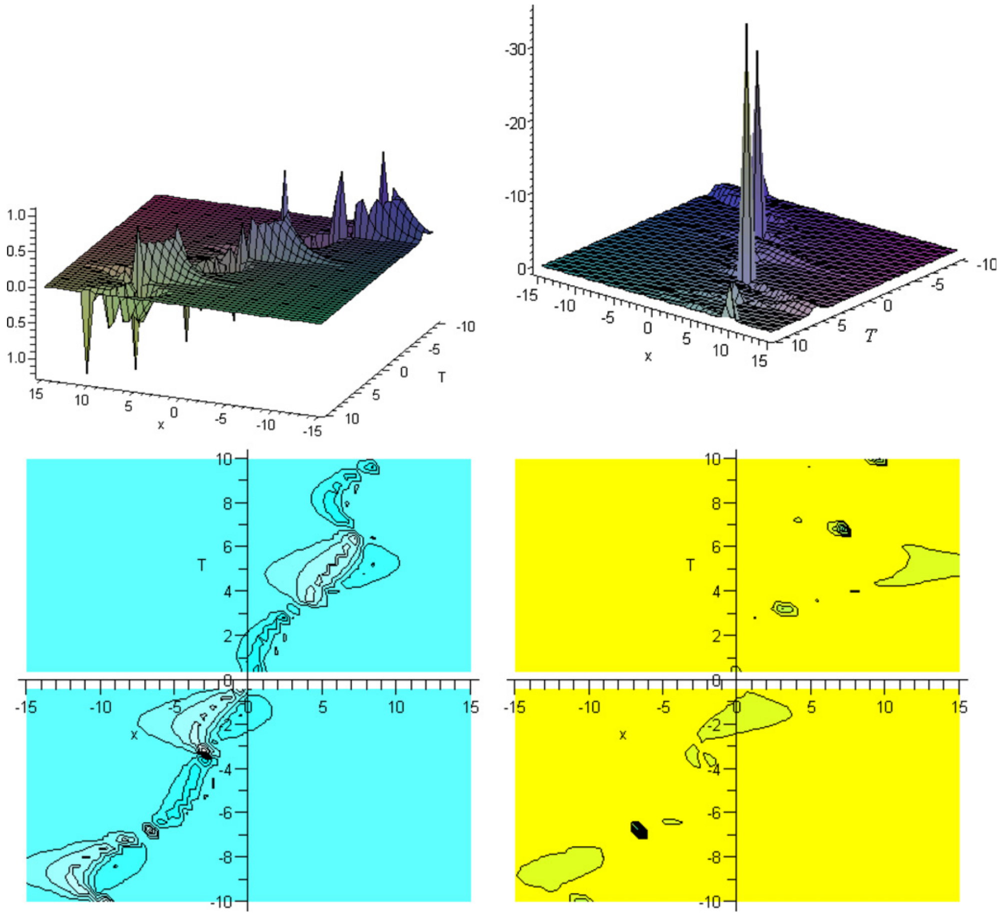

Figure 6. Imaginary part of the perturbed soliton and its contour plots with $\hat{\alpha} = 0.7$, $\beta = 0.1$, $\delta = 0.5$, $a_0 = 0.5$, $b_0 = 0.3$ and $k = 0.5$ (i) $\gamma = 0.0$ and (ii) $\gamma = 0.4$.

parameter is increased, the amplitude of the soliton decreases appreciably leading to the marginal reversal in the ferromagnetic medium.

4. Conclusions

In summary, we studied the effect of localized nonlinear inhomogeneity on the spin soliton of a classical continuum Heisenberg ferromagnetic spin chain. The effect of inhomogeneity was understood by carrying out a multiple perturbation analysis on an inhomogeneous generalized higher order NLS equation. We solved the coupled evolution equations for the velocity and amplitude of the soliton for nonlinear inhomogeneity in the form of a localized hyperbolic function using the Jacobi elliptic function method. The evolution of the amplitude and velocity of the soliton leads to magnetization reversal via the flipping of solitons in the ferromagnetic medium. In the presence of localized inhomogeneity the soliton undergoes curious changes and shows dramatic turns as the velocity and amplitude of the soliton change, periodically. This switching of behaviour of soliton is also studied and verified numerically through the fourth-order Runge–Kutta method. Thus, the nonlinear inhomogeneity acts as a good candidate for inducing the magnetization reversal through flipping of solitons. Finally, we have also constructed the perturbed soliton solutions. The above spin soliton flipping phenomenon which leads to magnetization reversal in a ferromagnetic medium is expected to have potential applications in magnetic memories and recording.

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Appendix A.

$$\alpha_1\alpha_2 b_{-1}^5 = 0, \quad -3\alpha_1\alpha_2 a_1^4 b_0 = 0, \quad 2\alpha_1^2 b_{-1}^5 - 3\alpha_2^2 b_{-1}^5 = 0, \quad 5\alpha_1\alpha_2 b_{-1}^4 b_0 + 8\alpha_1^2 b_{-1}^4 a_0 = 0,$$

$$10\alpha_1^2 b_{-1}^4 b_0 - 15\alpha_2^2 b_{-1}^4 b_0 = 0, \quad 20\alpha_1^2 b_{-1}^3 b_0^2 - 30\alpha_2^2 b_{-1}^3 b_0^2 - 10\alpha_1^2 b_{-1}^3 a_0^2 + 2b_{-1} = 0,$$

$$8\alpha_1^2 b_{-1}^4 a_1 + 22\alpha_1\alpha_2 a_0^2 b_{-1}^3 + 32\alpha_1^2 b_{-1}^3 b_0 a_0 + 10\alpha_1\alpha_2 b_{-1}^3 b_0^2 = 0,$$

$$66\alpha_1\alpha_2 a_0^2 b_{-1}^2 b_0 + 10\alpha_1\alpha_2 b_{-1}^2 b_0^3 + 48\alpha_1^2 b_{-1}^2 b_0^2 a_0 + 32\alpha_1^2 b_{-1}^3 b_0 a_1 + 44\alpha_1\alpha_2 a_0 a_1 b_{-1}^3 \\ + 12\alpha_2^2 a_0^3 b_{-1}^2 - 4\alpha_1^2 a_0^3 b_{-1}^2 = 0,$$

$$132\alpha_1\alpha_2 a_0 a_1 b_{-1}^2 b_0 + 22\alpha_1\alpha_2 a_1^2 b_{-1}^3 - 3\alpha_1\alpha_2 a_0^4 b_{-1} - 8\alpha_1^2 a_0^3 b_{-1} b_0 + 48\alpha_1^2 b_{-1}^2 b_0^2 a_1 \\ + 32\alpha_1^2 b_{-1} b_0^3 a_0 + 5\alpha_1\alpha_2 b_{-1} b_0^4 - 12\alpha_1^2 a_0^2 a_1 b_{-1}^2 + 24\alpha_2^2 a_0^3 b_{-1} b_0 \\ + 36\alpha_2^2 a_0^2 a_1 b_{-1}^2 + 66\alpha_1\alpha_2 a_0^2 b_{-1} b_0^2 = 0,$$

$$-4\alpha_1^2 a_0^3 b_0^2 + \alpha_1\alpha_2 b_0^5 + 8\alpha_1^2 b_0^4 a_0 + 12\alpha_2^2 a_0^3 b_0^2 - 24\alpha_1^2 a_0^2 a_1 b_{-1} b_0 - 12\alpha_1^2 a_0 a_1^2 b_{-1}^2 \\ + 66\alpha_1\alpha_2 a_1^2 b_{-1}^2 b_0 + 22\alpha_1\alpha_2 a_0^2 b_0^3 + 132\alpha_1\alpha_2 a_0 a_1 b_{-1} b_0^2 - 12\alpha_1\alpha_2 a_0^3 a_1 b_{-1} \\ - 3\alpha_1\alpha_2 a_0^4 b_0 + 36\alpha_2^2 a_0 a_1^2 b_{-1}^2 + 72\alpha_2^2 a_0^2 a_1 b_{-1} b_0 + 32\alpha_1^2 b_{-1} b_0^3 a_1 = 0,$$

$$-4\alpha_1^2 a_1^3 b_{-1}^2 - a_1 k^2 - a_1 + 8\alpha_1^2 b_0^4 a_1 - 24\alpha_1^2 a_0 a_1^2 b_{-1} b_0 - 12\alpha_1^2 a_0^2 a_1 b_0^2 + 44\alpha_1\alpha_2 a_0 a_1 b_0^3 \\ + 12\alpha_2^2 a_1^3 b_{-1}^2 - 12\alpha_1\alpha_2 a_0^3 a_1 b_0 - 18\alpha_1\alpha_2 a_0^2 a_1^2 b_{-1} + 36\alpha_2^2 a_0^2 a_1 b_0^2 \\ + 72\alpha_2^2 a_0 a_1^2 b_{-1} b_0 + 66\alpha_1\alpha_2 a_1^2 b_{-1} b_0^2 = 0,$$

$$36\alpha_2^2 a_0 a_1^2 b_0^2 + 24\alpha_2^2 a_1^3 b_{-1} b_0 - 12\alpha_1\alpha_2 a_0 a_1^3 b_{-1} - 18\alpha_1\alpha_2 a_0^2 a_1^2 b_0 + 22\alpha_1\alpha_2 a_1^2 b_0^3 \\ - 12\alpha_1^2 a_0 a_1^2 b_0^2 - 8\alpha_1^2 a_1^3 b_{-1} b_0 = 0,$$

$$-4\alpha_1^2 a_1^3 b_0^2 + 12\alpha_2^2 a_1^3 b_0^2 + 2a_1 k^2 - 12\alpha_1\alpha_2 a_0 a_1^3 b_0 - 3\alpha_1\alpha_2 a_1^4 b_{-1} = 0,$$

$$-24\alpha_1\alpha_2 b_{-1}^2 a_0^3 - 30\alpha_1^2 b_{-1}^2 b_0 a_0^2 - 20\alpha_1^2 b_{-1}^3 a_0 a_1 + 20\alpha_1^2 b_{-1}^2 b_0^3 - 30\alpha_2^2 b_{-1}^2 b_0^3 = 0,$$

$$-b_{-1} k^2 - 15\alpha_2^2 b_{-1} b_0^4 - 9\alpha_2^2 a_0^4 b_{-1} + 10\alpha_1^2 b_{-1} b_0^4 - 10\alpha_1^2 b_{-1}^3 a_1^2 - 30\alpha_1^2 b_{-1} b_0^2 a_0^2 \\ - 60\alpha_1^2 b_{-1}^2 b_0 a_0 a_1 - 48\alpha_1\alpha_2 b_{-1} b_0 a_0^3 - 72\alpha_1\alpha_2 b_{-1}^2 a_0^2 a_1 - b_{-1} = 0,$$

$$-60\alpha_1^2 b_{-1} b_0^2 a_0 a_1 - 72\alpha_1\alpha_2 b_{-1}^2 a_0 a_1^2 - 144\alpha_1\alpha_2 b_{-1} b_0 a_0^2 a_1 - 36\alpha_2^2 a_0^3 a_1 b_{-1} \\ - 10\alpha_1^2 b_0^3 a_0^2 - 9\alpha_2^2 a_0^4 b_0 - 3\alpha_2^2 b_0^5 + 2\alpha_1^2 b_0^5 - 24\alpha_1\alpha_2 b_0^2 a_0^3 - 30\alpha_1^2 b_{-1}^2 b_0 a_1^2 = 0,$$

$$-54\alpha_2^2 a_0^2 a_1^2 b_{-1} - 72\alpha_1\alpha_2 b_0^2 a_0^2 a_1 - 36\alpha_2^2 a_0^3 a_1 b_0 - 20\alpha_1^2 b_0^3 a_0 a_1 - 24\alpha_1\alpha_2 b_{-1}^2 a_1^3 - 30\alpha_1^2 b_{-1} b_0^2 a_1^2 - 144\alpha_1\alpha_2 b_{-1} b_0 a_0 a_1^2 = 0,$$

$$-48\alpha_1\alpha_2 b_{-1} b_0 a_1^3 - 54\alpha_2^2 a_0^2 a_1^2 b_0 - 36\alpha_2^2 a_0 a_1^3 b_{-1} - 10\alpha_1^2 b_0^3 a_1^2 - 72\alpha_1\alpha_2 b_0^2 a_0 a_1^2 = 0,$$

$$-9\alpha_2^2 a_1^4 b_{-1} - 36\alpha_2^2 a_0 a_1^3 b_0 - 24\alpha_1\alpha_2 b_0^2 a_1^3 = 0, \qquad -9\alpha_2^2 a_1^4 b_0 = 0.$$

Appendix B.

$$\begin{aligned}
\phi_{11}\int_{-\infty}^{\theta}\phi_{12}\Re F_1(\hat{q}_0)\,\mathrm{d}\theta &= \left[\left(\frac{1}{2}\hat{\alpha}\xi^2 - \frac{1}{2}\hat{\alpha}\eta^2 - \gamma\xi\eta\right)\Theta + \frac{3}{4\eta}(\xi\theta_{0T} + \sigma_{0T})\right]\operatorname{sech}\eta\Theta \\
&+ \left[(2\gamma\xi\eta - \hat{\alpha}\xi^2 + 2\hat{\alpha}\eta^2)\Theta - \frac{3}{4\eta}(\xi\theta_{0T} + \sigma_{0T})\right]\operatorname{sech}^3\eta\Theta \\
&+ \left[\frac{1}{2}\hat{\alpha}\xi^2 - \gamma\xi\eta - \frac{5}{2}\hat{\alpha}\eta^2\right]\Theta\operatorname{sech}^5\eta\Theta + \hat{\alpha}\eta^2\Theta\operatorname{sech}^7\eta\Theta \\
&+ \left[\frac{1}{4}\xi_T\Theta^2 - \frac{1}{2}(\xi\theta_{0T} + \sigma_{0T})\Theta - \frac{2}{3}\hat{\alpha}\eta\ln\cosh\eta\Theta\right]\operatorname{sech}\eta\Theta\tanh\eta\Theta \\
&+ \left[\frac{3}{4}(\xi\theta_{0T} + \sigma_{0T})\Theta - \frac{3}{4}\xi_T\Theta^2 + \frac{1}{2\eta}\hat{\alpha}\xi^2 - \frac{11}{6}\hat{\alpha}\eta - \gamma\xi\right]\operatorname{sech}^3\eta\Theta\tanh\eta\Theta \\
&+ \hat{\alpha}\eta\operatorname{sech}^5\eta\Theta\tanh\eta\Theta,
\end{aligned}$$

$$\begin{aligned}
\phi_{12}\int_{-\infty}^{\theta}\phi_{11}\Re F_1(\hat{q}_0)\,\mathrm{d}\theta &= \left[\left(\frac{1}{2\eta}\xi_T + \frac{1}{2}\hat{\alpha}\xi^2 - \frac{1}{2}\hat{\alpha}\eta^2 - \gamma\xi\eta\right)\Theta + \frac{1}{4\eta}(\xi\theta_{0T} + \sigma_{0T})\right]\operatorname{sech}\eta\Theta \\
&+ \left[(2\gamma\xi\eta - \hat{\alpha}\xi^2 + 2\hat{\alpha}\eta^2)\Theta - \frac{3}{4\eta}(\xi\theta_{0T} + \sigma_{0T})\right]\operatorname{sech}^3\eta\Theta \\
&+ \left[\frac{1}{2}\hat{\alpha}\xi^2 - \gamma\xi\eta - \frac{5}{2}\hat{\alpha}\eta^2\right]\Theta\operatorname{sech}^5\eta\Theta + \hat{\alpha}\eta^2\Theta\operatorname{sech}^7\eta\Theta \\
&+ \left[\frac{4}{3}\gamma\xi - \frac{2}{3\eta}\hat{\alpha}\xi^2 - \frac{3}{4\eta^2}\xi_T + \hat{\alpha}\eta\right]\operatorname{sech}\eta\Theta\tanh\eta\Theta \\
&+ \left[\frac{3}{4}(\xi\theta_{0T} + \sigma_{0T} - \xi_T\Theta)\Theta + \frac{1}{2\eta}\hat{\alpha}\xi^2 - \frac{11}{6}\hat{\alpha}\eta - \gamma\xi\right]\operatorname{sech}^3\eta\Theta\tanh\eta\Theta \\
&+ \hat{\alpha}\eta\operatorname{sech}^5\eta\Theta\tanh\eta\Theta + \frac{1}{6\eta^2}\left[\frac{3}{2}\xi_T + \hat{\alpha}\eta\xi^2 - \hat{\alpha}\eta^3 - 2\gamma\xi\eta^2\right]\sin\eta\Theta,
\end{aligned}$$

$$\begin{aligned}
\psi_{11}\int_{-\infty}^{\theta}\psi_{12}\Im F_1(\hat{q}_0)\,\mathrm{d}\theta &= \left[\frac{1}{4}\eta_T\Theta^2 + \frac{1}{2}\eta\Theta\Theta_T\right]\operatorname{sech}\eta\Theta \\
&- \frac{1}{4}\left[\eta_T\Theta^2 + \eta\Theta\Theta_T - \frac{4}{3}(\gamma\eta + 2\hat{\alpha}\xi)\right]\operatorname{sech}^3\eta\Theta \\
&- \left[\left(\frac{1}{6}\gamma\eta^2 + \frac{1}{3}\hat{\alpha}\xi\eta + \frac{1}{2}\gamma\xi^2 + \frac{1}{2\eta}\eta_T\right)\Theta + \frac{1}{4}\Theta_T\right]\operatorname{sech}\eta\Theta\tanh\eta\Theta \\
&- \frac{1}{3}(\gamma\eta^2 + 2\hat{\alpha}\eta\xi)\Theta\operatorname{sech}^3\eta\Theta\tanh\eta\Theta + \frac{2}{3}(\gamma\eta + 2\hat{\alpha}\xi)\operatorname{sech}\eta\Theta\ln\cosh\eta\Theta,
\end{aligned}$$

$$
\begin{aligned}
\psi_{12}\int_{-\infty}^{\theta}\psi_{11}\Im F_1(\hat{q}_0)\,\mathrm{d}\theta = &-\left[\frac{1}{3}\gamma\eta+\frac{2}{3}\hat{\alpha}\xi\right]\operatorname{sech}\eta\Theta \\
&-\left[\frac{1}{4}\eta_T\Theta^2+\frac{1}{4}\eta\Theta\Theta_T-\frac{1}{3}(\gamma\eta+2\hat{\alpha}\xi)\right]\operatorname{sech}^3\eta\Theta \\
&-\left[\left(\frac{1}{6}\gamma\eta^2+\frac{1}{3}\hat{\alpha}\xi\eta+\frac{1}{2}\gamma\xi^2\right)\Theta+\frac{1}{4}\Theta_T\right]\operatorname{sech}\eta\Theta\tanh\eta\Theta \\
&-\frac{1}{3}(\gamma\eta^2+2\hat{\alpha}\eta\xi)\Theta\operatorname{sech}^3\eta\Theta\tanh\eta\Theta \\
&-\left[\frac{1}{4\eta^2}\eta_T+\frac{1}{3}\hat{\alpha}\xi+\frac{1}{6}\gamma\eta+\frac{1}{2\eta}\gamma\xi^2\right]\sinh\eta\Theta\tanh\eta\Theta.
\end{aligned}
$$

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